Harvard Undergraduate Science Olympiad Invitational 2020

Astronomy C

Answer Key

Questions? Contact us at awli@college.harvard.edu and ashernoel@college.harvard.edu

Directions: Unless otherwise stated, Q1-7 sub-questions are worth two points each.

1 Answer Key A: Questions 1-4

1. (a) MACS J0717.5+3745 (+1), Radio (+1)
   (b) 3C 273 (+1), Virgo (+1)
   (c) JKCS 041 (+1), X-ray (+1)
   (d) Bullet Cluster (+1), anything related to dark matter (+1)
   (e) SN UDS10Wil (+1), 1.914 (+1)
   (f) GRB 150101B (+1), 2MASX J12320498-1056010 (+1)
   (g) MACS J1149.5+2223 (+1), 2007 (+1)
   (h) NGC 2623 (+1), SABcd (+1)
   (i) H2356-309(+1), WHIM (+1)
   (j) M87 (+1), Event Horizon Telescope (+1)
2. (a) "RV Tau" OR "RV Tauri"
   (b) "Helium" OR "He"
   (c) RRc
   (d) -0.5 - -0.7
   (e) "Type Ia" OR "Ia"
   (f) "None" OR "NA" OR "No CCOs/remnants"
   (g) "AM Her" OR "AM Herculis" OR "Polars"
   (h) "no accretion disk" OR "strong magnetic field"
   (i) "ZZ Ceti" OR "DAV"
   (j) Gravity wave pulsations
3. (a) 1.9-2.0
   (b) 0.2-0.4
   (c) Matter
   (d) 24-25
   (e) 1.1-1.2
   (f) 25-27
   (g) 2-7 e 3 Mpc
   (h) blue line with $\Omega_m = \Omega_0 = 1$
   (i) 4000-6000 MeV/m3
   (j) H I, H II (+2)
   (k) >1.5 M_{sun} (+1) Rotating can support a super-Chandrasekhar white dwarf (+1)
4. (a) Qualitative difference: higher redshift forests are flatter, have less spectral lines, have to travel through an opaque medium, or etc. (+2).
   (b) Lyman-alpha absorption lines involve the transition from the first to the second (+1) sub-level of neutral hydrogen (+1).
   (c) 5.8-6.2
   (d) 0.13-0.15
   (e) 722 Myr-1.15 Gyr.
   (f) (2 points maximum!) This supports the bottom-up (+1) mode; OR It accounts for the early formation of quasars and galaxies (+1); OR Cold dark matter in the bottom up model would promote the formation of these structures earlier than hot dark matter (+1).

(g) any mention of "it supports cold matter" (+1) because cold matter allows for the faster assembly of structure becasue hot dark matter would result in free streaming which would have removed primordial density fluctuations (+1).

(h) It does not comment on the presence of dark energy or the value of the cosmological constant (+1), but it does support cold dark matter (CDM) (+1)

2 Answer Key B: Questions 5-7

5. (a) 275 km/s
   (b) Award two points for mentioning any one of the following, up to a maximum of two points:
      i. Graphite
      ii. Polycylic Aromatic Hydrocarbons (PAH)
      iii. Amorphous Carbon
   (c) Sc
   (d) -23 - -24
   (e) -23.5 - -24.5
   (f) 3 - 4 $\times 10^{11} L_{\odot}$
   (g) 6 - 7
   (h) Green
   (i) 2 - 3
   (j) 60 - 90 Mpc
6. (a) s-process
   (b) Faber-Jackson relation
   (c) supermassive black hole
   (d) (i) E (+1) (ii) N (+1)
   (iii) K (+1) (iv) K (+1)
   (v) ”NA” or ”None” (+1) (vi) ”NA” OR ”None” (+1)
   (e) (i) Galactic Cannibalism (+1) (ii) Hubble Flow (+1)
   (iii) Thorne-Zytkow Objects (+1) (iv) Asteroseismology (+1)
   (f) (i) Pair instability (+1) (ii) ”None” OR ”NA” OR ”No Remnant” (+1)
7. (a) Award one point for mentioning any of the following, up to a maximum of two points:
      i. Forbidden
      ii. low density
   (b) (i) Blue (ii) Red (iii) Blue
   (c) ”Kennicutt-Schmidt Law/Relation” OR “Schmidt Relation/Law”
   (d) Filled Square
   (e) 0.016 - 0.026
   (f) 5.5 - 7.5 $\times 10^3$ km/s
   (g) 50 - 65 km/s/Mpc
   (h) 15 - 20 Gyr

3 Answer Key C: Questions 8-10

8. (a) *Solution.* In this problem, $\vec{r} = (r\cos\theta, r\sin\theta)$ and $\vec{a} = (a, 0)$. Then we have the tidal force given by

$$\vec{F}_t = G_2 Mm \frac{\vec{a} - \vec{r}}{|\vec{a} - \vec{r}|^2} - G_2 Mm \frac{\vec{a}^2}{|\vec{a}|^2} \tag{1}$$

We can simplify this equation by Taylor expanding the first term in powers of $\vec{r}$ and using the fact that $\vec{a} \gg \vec{r}$. Then the above equation becomes:

$$\begin{aligned}
\vec{F}_t &= G_2 Mm \frac{\vec{a} - \vec{r}}{(\vec{a} - \vec{r}) \cdot (\vec{a} - \vec{r})} - G_2 Mm \frac{\vec{a}^2}{|\vec{a}|^2} && (2)\\
&\approx G_2 Mm\vec{a}(|\vec{a}|^2 - 2\vec{a} \cdot \vec{r})^{-1} - G_2 Mm \frac{\vec{a}}{|\vec{a}|^2} - G_2 Mm \frac{\vec{r}}{|\vec{a}|^2} && (3)\\
&\approx G_2 Mm \frac{\vec{a}}{|\vec{a}|^2}(1 - 2\vec{a} \cdot \vec{r}/|\vec{a}|^2)^{-1} - G_2 Mm \frac{\vec{a}}{|\vec{a}|^2} - G_2 Mm \frac{\vec{r}}{|\vec{a}|^2} && (4)\\
&\approx G_2 Mm \frac{\vec{a}}{|\vec{a}|^2}(1 + 2\vec{a} \cdot \vec{r}/|\vec{a}|^2 + \cdots)^{-1} - G_2 Mm \frac{\vec{a}}{|\vec{a}|^2} - G_2 Mm \frac{\vec{r}}{|\vec{a}|^2} && (5)\\
&\approx G_2 Mm \left(2 \cdot \frac{\vec{a}(\vec{a} \cdot \vec{r})}{|\vec{a}|^4} - \frac{\vec{r}}{|\vec{a}|^2}\right) = \boxed{\frac{G_2 Mm}{a^2}\left(2\hat{a}(\hat{a} \cdot \vec{r}) - \vec{r}\right)} && (6)
\end{aligned}$$

In this derivation, we first expand the first term. Observe that the $\vec{r} \cdot \vec{r}$ in the denominator vanishes because $\vec{a} \gg \vec{r}$ and the numerator becomes $\vec{a}$, ie. $\vec{a} - \vec{r} \approx \vec{a}$. We can then Taylor expand the first term around $\vec{r} = 0$ and take the significant terms. Combining like terms yields our final answer as desired.

Plugging in $\vec{r} = (r\cos\theta, r\sin\theta)$ and $\vec{a} = (a, 0), \hat{a} = (1, 0)$, our tidal force becomes:

$$\boxed{\frac{G_2 Mm}{a^2}(r\cos\theta, -r\sin\theta)} \tag{7}$$

□

9. (a) *Solution.* Using the relationships given above, we can derive the following relationship for force:

$$\vec{F} = \vec{\omega} \times m\vec{v} = \vec{\omega} \times m\left(\vec{\omega} \times \vec{r}\right) \tag{8}$$

where $\vec{r}$ for the force is defined by the vector from the center of rotation/rotational axis to the blocks and $\vec{\omega}$ is the angular velocity vector given in the problem. Moreover, observe that immediately after disintegration (at time $t = 0$), the angular velocity of each resulting component remains unchanged.

Then solving for the four masses in the 1234-frame, we have

$$\vec{F}_1 = ml\omega_0^2\hat{z} \qquad \vec{F}_2 = -ml\omega_0^2\hat{z} \tag{9}$$

$$\vec{F}_3 = ml\omega_0^2\hat{z} \qquad \vec{F}_4 = -ml\omega_0^2\hat{z} \tag{10}$$

And for the 56-dumbbell, we have

$$\vec{F}_5 = ml\omega_0^2\hat{z} \qquad \vec{F}_6 = -ml\omega_0^2\hat{z} \tag{11}$$

The torques on all the masses is simplay zero after disintegration because all components are rotating about their principal axes — they require no torques to continue rotating. Mathematically, we can consider the change in angular momentum. We have

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{\vec{L}_{system}}{I_\perp} \times \vec{L}_{block} \tag{12}$$

where

$$\vec{L} \equiv \vec{\vec{\mathcal{I}}}\vec{\omega} \tag{13}$$

and $\vec{\vec{\mathcal{I}}}$ is the **inertia tensor**. This is simply a result of the conservation of angular momentum. As the blocks rotate, the angular momenta of the blocks does something crazy (since $\vec{v}$ is always changing), while the angular momentum of the components will be conserved. In particular, the angular momenta of the blocks is rotating around the angular momentum of the component (ie. the frame or dumbbell). We can again use the cross product in our calculation. Evaluating this above equation will yield

$$\vec{\tau}_1 = \vec{\tau}_2 = \vec{\tau}_3 = \vec{\tau}_4 = \vec{\tau}_5 = \vec{\tau}_6 = 0. \tag{14}$$

Alternatively, upon inspection, we can observe that $\vec{L}_{system}$ and $\vec{L}_{block}$ point in the same direction, which means the cross product vanishes as desired. □

(b) *Solution.* (Qualitative answer). The 1234-frame will remain unmoved and continue rotation in space. The 56-dumbbell will fly off with some tangential velocity and continue rotation with angular velocity $\vec{\omega}$.

(Quantitative solution.) We can model the system using the rotation matrix in the plane perpendicular to $\vec{\omega}$. In this case, the objects will rotate in the $x = -y$ plane with an angular frequency $\|\vec{\omega}\|$. This rotation matrix R is given by

$$R = \begin{pmatrix} \cos\left(\frac{\omega_0 t}{\sqrt{2}}\right)^2 & \sin\left(\frac{\omega_0 t}{\sqrt{2}}\right)^2 & \frac{1}{\sqrt{2}}\sin\omega_0 t\sqrt{2} \\ \sin\left(\frac{\omega_0 t}{\sqrt{2}}\right)^2 & \cos\left(\frac{\omega_0 t}{\sqrt{2}}\right)^2 & \frac{-1}{\sqrt{2}}\sin\omega_0 t\sqrt{2} \\ \frac{-1}{\sqrt{2}}\sin\omega_0 t\sqrt{2} & \frac{1}{\sqrt{2}}\sin\omega_0 t\sqrt{2} & \cos\omega_0 t\sqrt{2} \end{pmatrix} \tag{15}$$

This matrix describes the motion of any object rotating around the origin with angular velocity $\vec{\omega}$.

For the stationary 1234-frame after disintegration, we can dot R with the position vectors we used in **Problem 2.a** to determine the equations of motions of the rotation of our vectors around $\vec{\omega}$. We can then add the initial conditions (initial position of each block). Evaluating for each of the blocks, we have

$$\vec{s}_1(t) = \left(-\frac{l}{2} + \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, -\frac{l}{2} - \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, \frac{l}{2} + \frac{l\cos\sqrt{2}\omega_0 t}{2}\right) \tag{16}$$

$$\vec{s}_2(t) = \left(-\frac{l}{2} - \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, -\frac{l}{2} + \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, -\frac{l}{2} - \frac{l\cos\sqrt{2}\omega_0 t}{2}\right) \tag{17}$$

$$\vec{s}_3(t) = \left(\frac{l}{2} + \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, \frac{l}{2} - \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, \frac{l}{2} + \frac{l\cos\sqrt{2}\omega_0 t}{2}\right) \tag{18}$$

$$\vec{s}_4(t) = \left(\frac{l}{2} - \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, \frac{l}{2} + \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, -\frac{l}{2} - \frac{l\cos\sqrt{2}\omega_0 t}{2}\right) \tag{19}$$

For the 56-dumbbell, we can do the same thing as for the blocks in the 1234-frame but we must add in the tangential velocity of the dumbbell. We can again use

$$\vec{v}_{cm} = \vec{\omega} \times \vec{r}_{cm} \tag{20}$$

where $\vec{r}_{cm}$ is the vector from the origin to the center of mass of the dumbbell. Then our equations of motion for blocks 5 and 6 are

$$\vec{s}_5(t) = \left(\frac{l}{2} + \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, -\frac{l}{2} - \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, \frac{l}{2} - l\omega_0 + \frac{l\cos\sqrt{2}\omega_0 t}{2}\right) \tag{21}$$

$$\vec{s}_6(t) = \left(\frac{l}{2} - \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, -\frac{l}{2} + \frac{l\sin\sqrt{2}\omega_0 t}{2\sqrt{2}}, -\frac{l}{2} - l\omega_0 - \frac{l\cos\sqrt{2}\omega_0 t}{2}\right) \tag{22}$$

□

10. *Solution.* (a) Because we are in the ship's frame, the time measured by the clock is simply $\boxed{L/v_2}$.

(b) Because the astronaut's clock is *younger* than the clocks measured by the clocks on the train, the astronaut's clock measures a time

$$\boxed{\frac{L\sqrt{1 - v_2^2/c^2}}{v_2}}$$

The astronaut always moves with a speed v_2, the clocks in the ship frame measure the ticks of the astronaut's watch slowed down by a factor of $\gamma_2 = \frac{1}{\sqrt{1-v_2^2/c^2}}$.

□